

On Recent Claims Concerning the $R_h = ct$ Universe

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ABSTRACT

The $R_h = ct$ Universe is a Friedmann-Robertson-Walker (FRW) cosmology which, like Λ CDM, assumes the presence of dark energy in addition to (baryonic and non-luminous) matter and radiation. Unlike Λ CDM, however, it is also constrained by the equation of state (EOS) $p = -\rho/3$, in terms of the total pressure p and energy density ρ . One-on-one comparative tests between $R_h = ct$ and Λ CDM have been carried out using over 14 different cosmological measurements and observations. In every case, the data have favoured $R_h = ct$ over the standard model, with model selection tools yielding a likelihood $\sim 90 - 95\%$ that the former is correct, versus only $\sim 5 - 10\%$ for the latter. In other words, the standard model without the EOS $p = -\rho/3$ does not appear to be the optimal description of nature. Yet in spite of these successes—or perhaps because of them—several concerns have been published recently regarding the fundamental basis of the theory itself. The latest paper on this subject even claims—quite remarkably—that $R_h = ct$ is a vacuum solution, though quite evidently $\rho \neq 0$. Here, we address these concerns and demonstrate that all criticisms leveled *thus far* against $R_h = ct$, including the supposed vacuum condition, are unwarranted. They all appear to be based on incorrect assumptions or basic theoretical errors. Nevertheless, continued scrutiny such as this will be critical to establishing $R_h = ct$ as the correct description of nature.

Key words: cosmological parameters, cosmology: observations, cosmology: theory, gravitation

1 INTRODUCTION

One of the most basic FRW models, Λ CDM, assumes that the energy density of the Universe ρ contains matter ρ_m and radiation ρ_r , which we see directly, and an as yet poorly understand ‘dark’

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energy ρ_{de} , whose presence is required by a broad range of data including, and especially, Type Ia SNe (Riess et al. 1998; Perlmutter et al. 1999). In the concordance version of Λ CDM, dark energy is a cosmological constant Λ with an equation of state (EOS) $w_{\text{de}} \equiv w_{\Lambda} \equiv p_{\text{de}}/\rho_{\text{de}} = -1$. For the other two constituents, one simply uses the prescription $p_{\text{r}} = \rho_{\text{r}}/3$ and $p_{\text{m}} \approx 0$, consistent with a fully relativistic fluid (radiation) on the one hand, and a non-relativistic fluid (matter) on the other.

As the measurements continue to improve, however, the EOS $p = w\rho$, where $w = (\rho_{\text{r}}/3 - \rho_{\Lambda})/\rho$, appears to be creating some tension between theory and several observations. The concordance model does quite well explaining many of the data, but appears to be inadequate to explain all of the nuances seen in cosmic evolution and the growth of structure. For example, Λ CDM cannot account for the general uniformity of the CMB across the sky without invoking an early period of inflated expansion (Guth 1981; Linde 1982), yet the latest observations with *Planck* (Ade et al. 2013) suggest that the inflationary model may be in trouble at a fundamental level (Ijjas et al. 2013, 2014; Guth et al. 2013). And insofar as the CMB fluctuations measured with both WMAP (Bennett et al. 2003) and *Planck* are concerned, there appears to be some inconsistency between the predicted and measured angular correlation function (Copi et al. 2009, 2013; Melia 2014a; Bennett et al. 2013). There is also an emerging conflict between the observed matter distribution function, which is apparently scale-free, and that expected in Λ CDM, which has a different form on different spatial scales. The fine tuning required to resolve this difference led Watson et al. (2011) to characterize the matter distribution function as a ‘cosmic coincidence.’ It also appears that the predicted redshift-age relation in Λ CDM’s may not be consistent with the growth of quasars at high redshift (Melia 2013a), nor the emergence of galaxies at high redshift (Melia 2014b).

There is therefore considerable interest in refining the basic Λ CDM model, or perhaps eventually replacing it if necessary, to improve the comparison between theory and observations. Over the past several years, we have been developing another FRW cosmology, known as the $R_{\text{h}} = ct$ Universe, that has much in common with Λ CDM, but includes an additional ingredient motivated by several theoretical and observational arguments (Melia 2007; Melia & Abdelqadr 2009; Melia & Shevchuk 2012). Like Λ CDM, it also adopts the equation of state $p = w\rho$, with $p = p_{\text{m}} + p_{\text{r}} + p_{\text{de}}$ and $\rho = \rho_{\text{m}} + \rho_{\text{r}} + \rho_{\text{de}}$, but goes one step further by specifying that $w = (\rho_{\text{r}}/3 + w_{\text{de}}\rho_{\text{de}})/\rho = -1/3$ at all times. Some observational support for this constraint is provided by the fact that an optimization of the parameters in Λ CDM yields a value of w averaged over a Hubble time equal to $-1/3$ within the measurement errors. That is, though $w = (\rho_{\text{r}}/3 - \rho_{\Lambda})/\rho$ in Λ CDM cannot be equal to

$-1/3$ from one moment to the next, its value averaged over the age of the Universe¹ is equal to what it would have been in $R_h = ct$.

But there are good reasons to believe that w must in fact always be equal to $-1/3$ when one uses the FRW metric to describe the cosmic spacetime. This metric is founded on the Cosmological principle and Weyl’s postulate, which together posit that the Universe is homogeneous and isotropic (at least on large, i.e., > 100 Mpc, spatial scales), and that this high degree of symmetry must be maintained from one time slice to the next. Weyl’s postulate requires that every proper distance R in this spacetime be the product of a universal function of time $a(t)$ (the expansion factor) and a comoving distance r . As shown in Melia (2007) and Melia & Shevchuk (2012), the Misner-Sharp mass, given in terms of ρ and proper volume $4\pi R^3/3$ (Misner & Sharp 1964), defines a gravitational radius $R_h = ct$ for the Universe coincident with the better known Hubble radius $\equiv c/H$, where $H \equiv \dot{R}/R$ is the Hubble constant. Given its definition, $R_h = ct$ must itself be a proper distance, which trivially leads to the constraint $R_h = ct$, consistent with an EOS $p = -\rho/3$ (see also Melia & Abdelqader 2009). As further discussed in Melia (2007) and Melia & Abdelqader (2009), the corollary to Birkhoff’s theorem, which is of course valid in general relativity, provides additional justification—and a more pedagogical understanding—for defining a spherical *proper* volume in which to calculate the Misner-Sharp mass. Claims made to the contrary by Bilicki & Seikel (2012) and Mitra (2014) are simply incorrect, and stem from these authors’ misunderstanding of the use of Birkhoff’s theorem and its corollary (see also Weinberg 1972).

To test whether in fact the EOS $p = -\rho/3$ is maintained from one moment to the next, we have carried out an extensive suite of comparative tests using Λ CDM and $R_h = ct$, together with a broad range of observations, from the CMB (Melia 2014a) and high- z quasars (Melia 2013a, 2014b) in the early Universe, to gamma ray bursts (Wei et al. 2013a) and cosmic chronometers (Melia & Maier 2013) at intermediate redshifts and, most recently, to the relatively nearby Type Ia SNe (Wei et al. 2014a). The total number of tests is much more extensive than this, and includes the use of time-delay gravitational lenses (Wei et al. 2014b), the cluster gas-mass fraction (Melia 2013), and the redshift dependent star-formation rate (Wei et al. 2014c), among others. In every case, model selection tools indicate that the likelihood of $R_h = ct$ being correct is typically $\sim 90 - 95\%$ compared with only $\sim 5 - 10\%$ for Λ CDM. And perhaps the most important distinguishing

¹ It is not difficult to demonstrate this result. One simply assumes the WMAP values for the parameters in Λ CDM and calculates $w(t)$ as a function of cosmic time from the various contributions to p and ρ due to radiation, matter, and a cosmological constant. Then averaging $w(t)$ over a Hubble time, one finds that $\langle w \rangle \approx -0.31$. See the introductory discussion in Melia (2007) and Melia & Abdelqader (2009) and, especially, the more complete description in Melia (2009), particularly figure 1 in this paper.

feature between these two cosmologies is that, whereas Λ CDM cannot survive without inflation, the $R_h = ct$ Universe does not need it in order to avoid the well-known horizon problem (Melia 2013b).

Yet in spite of the compelling support provided for $R_h = ct$ by the observations, several authors have questioned the validity of this theory. The earlier claims made by Bilicki & Seikel (2012) have already been fully addressed in Melia (2012b), Melia & Maier (2013), and Wei et al. (2014a), so we will not revisit them here. Similarly, the criticisms made by van Oirschot et al. (2010) and Lewis (2012) concerning the definition and use of R_h are simply due to their improper use of null geodesics in FRW, a full accounting of which was published in Bikwa et al. (2012) and Melia (2012a). In this paper, we focus on the two most recent claims made concerning the $R_h = ct$ Universe: (1) that this cosmology is static and merely represents another vacuum solution (Mitra 2014), and (2) that the equation of state in $R_h = ct$ is inconsistent with $p = -\rho/3$, thus ruining the elegant, high-quality fits to the data (Lewis 2013). We will address these two concerns in §§ 2 and 3, respectively, and end with some concluding remarks in § 4.

2 ON MITRA'S CLAIM THAT $R_H = CT$ IS A VACUUM SOLUTION

Mitra (2014a, and references cited therein) has been trying for several years to confirm the validity and uniqueness of the $R_h = ct$ cosmology using the energy complex. This is the basis for the claim in his latest paper (Mitra 2014b) that since $R_h = ct$ is (according to him) a vacuum solution, all big bang models should be manifestations of the vacuum state as well.

His argument is based on a presumed demonstration that the $R_h = ct$ metric is static, for which he then concludes that $\dot{a} = 0$. And since the critical density is proportional to \dot{a} in the Friedmann equation, he makes the claim that $R_h = ct$ must therefore correspond to a vacuum spacetime.

But his analysis is incorrect for several reasons. First and foremost, it was proven several decades ago that there are exactly six—and *only six*—special cases of the FRW metric for which a transformation of coordinates is possible to render the metric coefficients $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$) independent of time x^0 . These correspond to solutions of the expansion factor $a(t)$ for which the spacetime curvature of the FRW metric is constant (Robertson 1929; Florides 1980; Melia 2012c, 2013c). As shown by Florides (1980) in his landmark paper, these special cases are (1) the Minkowski spacetime, (which is highly trivial), (2) the Milne Universe (with spatial curvature constant $k = -1$), (3) de Sitter space, (4) anti-de Sitter space, (5) an open Lanczos-like Universe,

and (6) the Lanczos Universe itself. The $R_h = ct$ Universe, with $a(t) \propto t$ and $k = 0$, is not one of them.

The spacetime curvature in $R_h = ct$ is *not constant* and the reason $\dot{a} = \text{constant}$ in this cosmology is not because $\rho = 0$ but, rather, because $\rho + 3p = 0$ —i.e., the ‘active mass’ is zero (Melia 2014c). Mitra’s derivation in § 3 of his paper is flawed because he assumes that the FRW metric can always be written in ‘Schwarzschild coordinates.’ But this too is incorrect because the transformed time T (measured from the big bang) is well defined in only a few special cases, as demonstrated several years ago by Melia & Abdelqader (2009). It is not possible to rewrite the FRW metric solely in terms of R and T in those cases where R can exceed R_h , which certainly happens at early times for cosmologies, such as Λ CDM and $R_h = ct$, with an initial singularity. (The de Sitter Universe is an obvious counter-example.)

For these reasons, it is simply wrong for Mitra to claim that the $R_h = ct$ Universe is merely another manifestation of the vacuum solution.

3 ON LEWIS’S CLAIM CONCERNING THE EOS IN $R_H = CT$

For reasons that are never made clear, Lewis (2013) assumes that a ‘pure’ $R_h = ct$ Universe is comprised of a single fluid (dark energy) with no matter, and an equation of state $w = w_{\text{de}} = -1/3$. He then makes the additional assumption that if matter were to be introduced into such a universe, it ought to be conserved separately from all the other constituents. Not only are these assumptions unnecessary, but there is actually no precedent for them either. In fact, they are incorrect from the outset.

As described above, this is not how the $R_h = ct$ Universe is set up. As noted earlier, the $R_h = ct$ Universe is Λ CDM with the additional constraint $w = -1/3$. This does not mean that $w_{\text{de}} = -1/3$, nor that $\rho_m = \rho_r = 0$. The models considered by Lewis should therefore be more aptly viewed as variants of Λ CDM, and we already know that in order for the standard model to have any hope of fitting the data, one must have $\Omega_m \equiv \rho_m(t_0)/\rho(t_0) \sim 0.27$ and (with analogous definitions) $\Omega_{\text{de}} \equiv \Omega_\Lambda \sim 0.73$, with the spatial flatness condition $\Omega_m + \Omega_r + \Omega_\Lambda = 1$. It is hardly surprising, then, that the unusual models considered by Lewis do not fit the data. They are neither $R_h = ct$ nor the concordance model either.

Second, the assumption of a separately conserved matter field is not used in FRW cosmologies, and is certainly not valid over the age of the Universe in Λ CDM. There is therefore no precedent for imposing it on $R_h = ct$ either. For example, Λ CDM invokes the idea that matter in the early

Universe was created and annihilated, exchanging its energy density with that of the radiation field (and possibly other fields that may emerge from extensions of the standard model of particle physics). In Λ CDM, matter may be separately conserved today if interactions such as these are currently inconsequential. However, we don't even know if dark matter is self-interacting, or if it decays. So matter could not have been separately conserved in the early Universe; it may not even be so conserved today, and may in fact never be conserved if its interactions with other energy fields continue indefinitely into the future. What we can say for sure in the case of $R_h = ct$ is that in order for the equation of state to be maintained at $w = -1/3$, the various constituents must adjust their relative densities via particle-particle interactions. But there is nothing mysterious about a situation such as this, in which the internal 'chemistry' of a system is controlled by external or global physical constraints. We do the same thing in the standard model when we force the temperature to obey a fixed functional dependence $T(z)$ on the redshift, and then require all the particle species to find their equilibrium through the various forces and interactions they experience with other components. In situations such as this, it is important to remember that particle numbers are not conserved, and each particle type is subject to the pressure of other species, not just its own, so one cannot naively assume that each component evolves as an independent density. For example, it is not correct to assume that prior to recombination, when matter and radiation were in local thermodynamic equilibrium, the matter energy density scaled as $\rho_m \sim a^{-3}$ and the radiation as $\rho_r \sim a^{-4}$. These only apply when matter and radiation evolve independently of each other.

This analogy may appear to be over-reaching; after all, the spectrum of the CMB is a spectacular Planck function. But there are already several indicators, some circumstantial, that the condition $w = -1/3$ is also being maintained as the Universe expands. We have already alluded to the fact that $\langle w \rangle \approx -1/3$ when $w(t)$ is averaged over a Hubble time. Such an average can emerge only once, in the entire history of the Universe—unless w were always equal to $-1/3$. Otherwise, it would be an extraordinary coincidence for us to be living just at this moment, the only instant when we can see this happen. This condition is also suggested by model-independent measurements of the Hubble constant $H(z)$ (Melia & Maier 2013), which are most consistent with $w = -1/3$ (i.e., $a[t] \propto t$), which results in $H(z) = H_0(1+z)$. And a more substantial analysis of the cosmic equation of state yields a strong correlation between the inferred values of Ω_m and w_{de} when optimizing the parameters in Λ CDM to fit the data (Melia 2014d). This correlation predicts that $\Omega_m \approx 0.27$ when $w_{de} = -1$, while w_{de} must be closer to -1.1 if $\Omega_m \approx 0.31$. Interestingly, the first pair of values corresponds to the WMAP results (Bennett et al. 2003), while the latter pair corresponds to the best fit using the Planck measurements (Ade et al. 2013). This is still only circumstantial evidence

at best, but it does suggest that the optimization of the parameters in Λ CDM is always restricted by the condition $w = -1/3$.

4 CONCLUSION

In spite of the many successes Λ CDM has enjoyed in accounting for the cosmic expansion, many today would agree that the ever-improving measurements are starting to reveal some possible inconsistencies between its predictions and the latest observations. We have highlighted several of these areas and the need to evolve the standard model in order to address these potential problems.

The $R_h = ct$ Universe is essentially Λ CDM with one additional constraint—the total EOS $p = -\rho/3$. This condition, which is motivated by several observational and theoretical arguments, appears to solve many of the conflicts otherwise experienced by the standard model. As of today, every one-on-one comparison carried out between these two models has statistically favoured the former. It is difficult to argue against this rate of success.

Nevertheless, the development of $R_h = ct$ as a comprehensive description of nature is hardly complete, inviting several concerted efforts at challenging its fundamental basis. Such scrutiny is an essential component of any serious discussion concerning its viability. As of today, however, all the criticisms raised thus far appear to have been based on incorrect assumptions or flawed theoretical arguments. In this paper, we have discounted the two most recent claims, one having to do with the presumed vacuous nature of the $R_h = ct$ metric, and the second with its possibly inconsistent equation of state.

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